A very simple model of vaccine rollout interruptions

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At the time of writing, a number of national Covid-19 vaccine programmes are being suspended and/or re-adjusted in the face of concerns about rare but serious (sometimes fatal) adverse reactions to vaccines. This has posed some very difficult dilemmas, including how best to understand the consequences of a pause in a vaccine rollout.

We present a very simple epidemiological model of the transmission of an infectious agent leading to transient infection/infectiousness and immunity upon recovery. This model does not claim to be able to represent, with any quantitative fidelity, any complex real world Covid-19 scenario, but we believe it demonstrates a heuristic point about the potential impact of pauses in vaccine rollout.

In our model world, there are the following ‘state variables’:

- S: the proportion of the population which is still susceptible to infection
- I: the proportion of the population which is currently infected
- R: the proportion of the population which has been infected but is already recovered
- V: the proportion of the population which has been vaccinated.

The model equations are given by the following Ordinary Differential Equations (ODEs):

$$\frac{dS}{dt} = -\lambda(t) - KSI\exp(-\alpha(1-S))$$

$$\frac{dI}{dt} = KSI\exp(-\alpha(1-S)) - \gamma I$$

$$\frac{dV}{dt} = +\lambda(t)$$

$$\frac{dR}{dt} = \gamma I$$

Expressing that:

- No one enters or leaves the population.
- Transmission is via mass action mixing, attenuated by a term which is intended to mimic additional reduction in transmission with the depletion of susceptibles, over and above that implied by mass action alone.
- Recovery times are exponentially distributed.
- Immunity via vaccination and recovery is absolute (on the time scales under discussion).

The parameters are
- K - contact rate
- γ- recovery rate
- α- dimensionless parameter to dial strength of transmission damping with reduction in susceptibles

And we treat as time dependent
- $\lambda(t)$ - the rate of vaccination (fraction of total population per unit time)

This model is very simplistic in all the usual ways that textbook type ODE epidemiological transmission models are. In particular, there is no non-trivial transmission 'network'. We would argue, however, that by varying a mixture of the parameters and initial conditions, and, most importantly, the rate of vaccination, one can capture meaningful qualitative features about the interaction of a new vaccine programme with an impending epidemic wave.

We doubt that these equations admit an analytical solution, but It is a simple matter to code up these equations on one of many suitable software tools for manipulating equations of this kind. The key features which appear to us to be relevant to the question of vaccination programmes are:
- The well known positive feedback between vaccine coverage and collective immunity. When enough people are immune, the reproductive number dips below one, and everyone receives strong protection even without everyone having been vaccinated
- When vaccine coverage is increasing substantially during the course of an outbreak, the impact of vaccination is particularly sensitive to the precise start date and rate of vaccination. A small headstart, or a small increase in vaccination rate, can have an impressive effect on the outbreaks total 'attack rate' (fraction of the population which is infected by the end of the outbreak)
- SImilarly, a small pause introduced early into a vaccination programme, in the face of a looming outbreak, can have a disproportionate negative effect on the impact of the vaccination programme.

Naturally, it would take some doing to produce a model which captures a substantial part of the complexity of any actual Covid-19 outbreak, and we do not propose that our simple model should form the basis of any formal accounting to produce estimates of infections avoided, cost effectiveness, etc.

We do propose that decisions to pause or delay vaccinations should not be taken lightly when in the midst of an uncontrolled epidemic such as is currently being experienced in many contexts.